EBBINGHAUS' EXPLANATION OF BEATS.

I. M. BENTLEY and E. B. TITCHENER.

This paper grew out of a discussion between the writers of the theory of beats set forth by Ebbinghaus in his *Psychologie*. The discussion brought out the fact that certain aspects of the theory had been, if not differently understood, at least very differently weighted and estimated by two interested readers of the book. Further reading and discussion have led, first, to an agreement as to the main features of Ebbinghaus' exposition, and secondly to the following joint criticism.

The paper is confined specifically to the subject of its title. The writers desire to insist upon this point, as it constitutes their apology for the neglect of many articles, etc., reference to which might, in view of the general topic, be expected by the reader.

THE HELMHOLTZ THEORY OF TONAL SENSATIONS.

According to the revised version of the Helmholtz theory, the basilar membrane may be considered as a system of stretched strings, in which every individual string moves in relative independence of all the others.[1] If a simple tone is struck outside the ear, its stimulus throws a part of the basilar membrane into sympathetic vibration. That fibre vibrates most strongly whose proper motion corresponds most nearly to the movement of the external stimulus. From this fibre the vibrations extend on both sides, with rapidly diminishing strength, to the adjacent parts of the membrane. The sympathetic vibration thus set up "is still sensible for the interval of a semitone."[2]

Helmholtz applies Johannes Müller's doctrine of specific sensory energies to the fibres of the cochlear nerve: difference of tonal pitch is correlated with difference in the nerve fibre excited.[3] If, then, we follow the strict letter of the theory, we must suppose that the basilar fibres which vibrate on either side of the principal fibre, in any given case of tonal stimulation, move at the rate imposed on them by the stimulus, *i. e.*,

[1] *On the Sensations of Tone*, trs. by A. J. Ellis, 1895, 146 *b*.

[2] *Ibid.*, 144 *c*, 146 *c*, 147 *d*. The German word for 'sensible' is *merklich*. The context is physiological, and the term means 'large enough, physically, to be worth taking account of.'

[3] *Ibid.*, 148 *b*, *c*.

at the same rate as the principal fibre, but nevertheless mediate their own sensations. We have a strip or bundle of basilar fibres, all vibrating at the same frequency — though with rapidly lessening intensity as we leave the center of the strip, and all exciting the central organ in their own specific ways.[1]

There is a further point dealt with by Helmholtz in an Appendix,[2] which becomes of importance later in our discussion. Theoretically, *i. e.*, if we make certain assumptions with regard to the structure of the basilar membrane and draw their mathematical consequences, "every excitement of a bundle of transverse fibres by the fundamental tone must be accompanied by weaker excitements of the unevenly numbered harmonic undertones." These undertone fibres excite their own nerve fibres, and should accordingly, mediate their own sensations. "There is nothing of the kind observable in the ear." The reason is that "the appendages of the basilar membrane probably greatly impede the formation of tones with nodes."[3]

The physiological substrate of a simple tonal sensation is, therefore, threefold. We have (1) the strong sympathetic vibration of the basilar fibre whose proper tone lies nearest the tone of the stimulus. We have (2) the weaker sympathetic vibration of adjacent fibres. We have (3) the weak sympathetic vibration, with formation of nodes, of fibres carrying the odd-numbered harmonic undertones. The fibres of the two latter groups ought to arouse each their own proper tone. The undertone fibres, however, are so quickly damped that they are not represented in sensation. Helmholtz does not seem to have faced the difficulty that arises from the sympathetic vibration of the fibres adjoining the principal fibre, — the difficulty that these fibres must, in strictness, arouse their own tones, while only one single tone (the tone of the principal fibre) exists for sensation.

It may be said, at once, that this difficulty is not an insuperable objection to the Helmholtz theory. It may be overcome in various ways. Thus Stumpf, of whose views we shall have more to say presently, thinks on physiological and psy-

[1] The easiest way to envisage the Helmholtz theory is to think of it as a one-to-one correlation of basilar fibre and specific nerve fibre. And there is nothing seriously misleading in such envisagement. It should, however, be remembered that Helmholtz himself speaks in terms, not of the basilar fibres, but of Corti's arches and Corti's fibres: see, *e. g.*, *ibid.*, 147 *b*, *c*. He nowhere commits himself to the statement that the number of basilar fibres is a measure of the number of possible tone sensations.

[2] *Ibid.*, 411 *b*, *c*.

[3] It appears (unfortunately we have not the earlier editions of the *Tonempfindungen* at hand) that these considerations were suggested to Helmholtz by Riemann's *Musikalische Logik*, 1877.

chological grounds that an accommodation of the specific energies of the nervous structures, within certain narrow limits of stimulation, is not only possible but even highly probable.[1]

Ebbinghaus' Modifications.

Ebbinghaus accepts, without reserve, the fundamental idea of the Helmholtz theory.[2] In order, however, to meet certain difficulties of application, he proposes two modifications. (1) The first is directed against an extreme interpretation of the doctrine of specific energies. The cells of the cochlear nerves, or their central representatives, are so constituted as to mediate tonal sensations under the influence of rhythmical stimuli. Originally, Ebbinghaus says, any cell can mediate any sensation. But since the cell is connected with the basilar fibre, only a certain limited number of rhythms are, as a matter of fact, conducted to it. The rhythm that affects it most easily is, of course, the proper rhythm of its own basilar fibre; so that, by a process of physiological adaptation (*Gewöhnung*), it comes to respond most quickly and most strongly to this particular form of stimulus. It can, however, also respond, within certain limits, to other rhythms.[3]

Suppose, then, that a simple tonal stimulus is given. It sets in vibration a strip of the basilar membrane, a group of adjacent basilar fibres. These fibres vibrate, one and all, in the rhythm of the incoming stimulus. The corresponding cells mediate, not each its own sensation, but all the same sensation, *i. e.*, the sensation directly correlated with the stimulus. They do this because they still retain something of their primitive indifference of function: they are not so specifically adapted that they cannot take up rhythmical impulses which vary, in some small degree, from the rhythm of their proper fibre. The cell that is most nearly attuned to the stimulus arouses the strongest excitation; the neighboring cells repeat the same excitation, only in weaker form.

(2) Helmholtz had remarked that the fibres carrying certain harmonic undertones must vibrate, with the formation of nodes, on the operation of a given tonal stimulus. He supposed these fibres to mediate their own sensations; and accounted for the fact that their tones are not heard by the weight of the structures attached to the basilar membrane.

---

[1] *Tonpsychologie*, ii, 1890, 111 ff., with other references there given. At the time that the *Tonpsychologie* was written, Stumpf was an adherent of the Helmholtz theory. He has now given it up: see, *e. g.*, *Beitr. z. Akustik u. Musikwiss*, i, 1898, 18 *note*, 51 f. This fact should be borne in mind in reading the later divisions of the present paper.

[2] *Psychologie*, i, 1897, 316.

[3] *Ibid.*, 318 ff.

Ebbinghaus questions the soundness of this explanation; strings are capable of strong vibration even when very heavily loaded. He himself believes that the undertone fibres vibrate, with the formation of nodes. But he differs from Helmholtz in supposing that not the odd-numbered only, but all the undertone fibres, down to a certain limit, are involved; and, further, that the partial vibrations as such constitute the stimulus to the relatively indifferent cells. The cell is primarily adapted to the rhythm of its proper fibre. It is secondarily adapted, as we have seen, to the rhythms of adjacent fibres. It is also secondarily adapted to rhythms that are twice, three times, four times, etc., as rapid as the rhythm of its proper fibre. The undertone fibres consequently mediate precisely the same sensation as do the fibres of the strip set in direct vibration by the stimulus.[1]

An illustration will make the matter clear. Suppose that a tone of 600 vs. in the 1 sec. is sounded outside the ear. The effect upon the cochlea is (*a*) that the 600-fibre and its cell are thrown into strong vibration. At the same time (*b*) the neighboring fibres and cells, above and below, are thrown into weaker vibration, and so mediate the same excitation in weaker form. Further, (*c*) the 300-fibre vibrates at twice its natural rate, the 200-fibre at three times its natural rate, and so on. The cell of the 300-fibre accepts, takes up, the 600-rhythm, and mediates (if we may use the expression) the 600-sensation. The cell of the 200-fibre also accepts the 600-rhythm, and mediates, less strongly, the same sensation. These long fibres mediate the sensation proper to the shorter 600-fibre by reason of the fact that their cells are fairly well accustomed to vibrations two, three, etc., times as rapid as those normal to the fibres upon which they sit.

We have nothing to object to these modifications. It must only be remembered that Ebbinghaus makes them under stress of the psychological facts, and that they are both entirely hypothetical. Helmholtz gives one conjectural interpretation of the function of the cochlea; Ebbinghaus gives another. If, now, it is a question of cutting a physiological theory to suit the needs of psychology, we shall incline to accept the theory, whosesoever it may be, that without doing violence to the established facts of physiology best takes account of the results of introspection. On the whole, Ebbinghaus' 'adaptation' seems to us to offer a better working hypothesis than Helmholtz' 'specific energy.' But both theories alike contain too much of guess-work and analogical reasoning to allow us to strike a decision between them. The test, for the psychologist,

[1] *Ibid.*, 320.

lies just in their power to explain. The test applied to Ebbinghaus' theory in the present paper is its explanation of the phenomena of beats.

The Phenomena of Beats.

There can be no doubt that Helmholtz regarded beats, primarily, as *intensive* fluctuations of tonal sensation.[1] There can be no doubt, either, that he began his investigation with the slow beats formed when the two primary tones lie very near together upon the scale, and worked from these to the more rapid beats produced when the tones lie farther apart. Hence, beats appeared to him to be a very simple phenomenon, to be explained, as simply, by reference to the interference of vibrations. "Corti's arches,"[2] he says, "may be made to beat by two tones sufficiently near in pitch to set the same Corti's arches in sympathetic vibration at the same time. If, then, . . . the intensity of auditory sensation in the nerve fibres involved increases and decreases with the intensity of the elastic vibrations, the strength of the sensation must also increase and diminish in the same degree as the vibrations of the corresponding elastic appendages of the nerves."[3]

If we grant the introspective assumptions, this explanation is valid. Two neighboring tones are sounded. Each sets in vibration its own strip of the basilar membrane. These strips overlap; the same basilar fibres are affected by both stimuli, and there is interference. We therefore hear precisely what we should hear if there were no beats, except that the mass o sound is interrupted (or is subject to intensive fluctuation) at intervals corresponding to the difference in the vibration frequencies of the primary tones. The explanation is straightforward, and follows directly from the theory. The initial difficulty of the theory, the difficulty that a vibrating strip of the basilar membrane ought to give us a medley of different tonal sensations, of course remains.

Unfortunately, however, the facts as reported by introspection are less simple than Helmholtz supposed. We take Stumpf's account.

"(*a*) If I give two tones, about a semitone apart, in the middle region of the scale (*e. g.*, g^1-sharp and a^1 on the violin), I hear the two primary tones, but also, over and above these, a third tone which lies between them, somewhat nearer the lower than the higher. This third tone has a very soft coloring, and with keen attention is localized within the ear; it is this tone which beats, while the primary tones remain con-

[1] *Sensations of Tone*, 164 *c*, 165 *c*, *d*, 169 *a*. Stumpf, *Tonpsychologie*, ii, 450; *Beitr.*, 1, 4, *note*.
[2] Used in the sense explained *ibid.*, 147 *b*.
[3] *Ibid.*, 166 *c*, 167 *b*, *c*, 172 *a*, *b*.

stant. The two primary tones are, in my judgment, noticeably weakened, — more than is customary when two tones are sounded at the same time.

"(*b*) If I take tones that lie farther apart, in the same region of the scale (*e. g.*, g^1 and a^1), I do not hear any middle tone, but only the two primaries; and these two seem themselves to beat. If, however, I turn the attention more particularly to the one of them, this always appears to be the beating tone.

"(*c*) If, on the other hand, I take two tones that lie much nearer together than a musical semitone, so that they approximate the difference limen for simultaneous tones, I get one tone, and that beating. It is difficult to say whether it lies between the primaries." [1]

Stumpf's Explanations.

Stumpf seeks to account for these three observations in terms of the Helmholtz theory as modified by the hypothesis of physiological accommodation (accommodation of neighboring nerve fibres or their ganglion cells). If we grant Stumpf's presuppositions, the explanations are satisfactory. They are, in brief, as follows.

It is clear that Obs. (*c*)[2] is the typical observation required by the Helmholtz theory, and agrees with the observations made by Helmholtz himself. We need, therefore, spend no time upon it.

Obs. (*a*) demands more discussion. Stumpf supposes, with Helmholtz, that the two basilar strips, the g^1-sharp strip and the a^1 strip, partially overlap. We might, therefore, expect a priori to hear both the g^1-sharp and the a^1, and to hear both beating. What we actually hear is an intermediate beating tone, and the primaries sounding, smoothly but weakly, on either side. To explain the introspective phenomena, Stumpf appeals to his principle of nervous accommodation. There will be one intermediate nerve fibre, affected at equal strength by both forms of stimulation, for which the twofold "Erregungsweise am kräftigsten eintreten muss." It is then a simple extension of the principle of accommodation of nervous energies (*Bildung einer Gesammtenergie*) to assume that this intermediate nerve fibre or ganglion cell, giving its specific response under intensive stimulation, will constrain the neighboring fibres or cells in the direction of its own specific energy: so that the nervous structures excited fall, for purposes of sensation, into three groups, — an upper and a lower, replying specifically to the primary stimulations, and an intermediate, replying specifically to the intensive stimulation produced by

[1] Tonpsychologie, ii, 480 ff.

[2] *Ibid.*, 488 f.

the joint operation of the two primary stimuli. In other words, we hear an intermediate tone, which carries the beats, and the two outlying tones, not beating, and weakened by the withdrawal (so to say) of certain of their constituent nerve fibres or nerve cells to the specific functional activity of the central zone.[1]

If we grant the presuppositions, this explanation is entirely acceptable. Stumpf gives further and very ingenious explanations of the peculiar coloring of the third tone, its localization, its pitch, and so on. We need not here enter into these details.

We come to Obs. (*b*). In this case, there is, according to Stumpf, a very slight overlapping of the basilar strips. The double excitation is thus too weak to arouse the intermediate nerve fibres or cells to their specific response. They beat; but they beat in the tone of the singly excited fibre or cell group above or below them. We therefore hear only the two primary tones, sounding more strongly than in Obs. (*a*); and we hear them beating, for the reason that a certain portion of their physiological substrates is affected by both stimuli.[2] The shift of the beating tone with shift of attention is readily intelligible.

In fine, then, Stumpf is able to give, in terms of the Helmholtz theory and his own hypothesis of nervous accommodation, a coherent and adequate explanation of the three observations. His explanation of Obs. (*a*) is especially happy. It is, in effect, a compromise—if one may speak of a compromise before both of the extremes have been formulated—between Helmholtz' doctrine of specific energies, and Ebbinghaus' doctrine of the original functional indifference of the cochlear cells.[3]

EBBINGHAUS' EXPLANATIONS.

Ebbinghaus accepts Stumpf's description of the phenomena. After describing Obs. (*b*) and (*c*), he proceeds to Obs. (*a*), as follows. "Nur bei geringer Höhendifferenz der Töne, etwa bis zu einem Halbton, verhält es sich, worauf Stumpf erst aufmerksam machte, etwas anders. Man hört dann einen *zwischenliegenden* Ton als Träger der Schwebungen, während die beiden objektiv erzeugten Töne entweder abgeschwächt und ruhigbleibend daneben hörbar sind oder auch—bei den geringsten Höhendifferenzen—ganz verschwinden. Eine Erklärung dieser *Zwischentöne* oder *resultierenden Töne* (Melde) wird später gegeben werden."[4]

Turning to the passage referred to, we read as follows:

[1] *Ibid.*, 484 ff.
[2] *Ibid.*, 487 f.
[3] Nevertheless, as will presently appear, we have a possible physical explanation of the *Zwischenton*, acceptance of which would render the accommodation hypothesis unnecessary.
[4] *Psychologie*, i, 1897, 304 f.

"Diese [Schwebungen] sollen [nach der Helmholtz'schen Theorie] dadurch zustande kommen, dass bei dem gleichzeitigen Erklingen zweier hinreichend benachbarter Töne die mitschwingenden Gebiete der Basilarmembran nicht mehr ganz getrennt bleiben, sondern teilweise übereinandergreifen, und die Bewegungen nun an den mittleren Fasern miteinander interferieren. Das passt vortrefflich, wenn die beiden objektiven Töne sehr nahe bei einander liegen. Dann hört man, wie oben mitgeteilt, in der That den nach der Theorie zu erwartenden *Zwischenton* als Träger der Schwebungen." [1]

Could anything be less satisfactory? The *Zwischenton* is precisely what the Helmholtz theory does *not* explain; it is precisely what we should *not* expect from that theory. What the Helmholtz theory would give us, in its strict form, is a medley of tones, some of which (the intermediate tones) are beating. What it under no circumstances can give us is the actual fact of observation,—two steady primaries and one beating intermediate tone.[2]

We may add that this Obs. is not either explained by Ebbinghaus' own modification of the Helmholtz theory. If the nervous appendages of the two basilar strips are 'adapted' to take up the two primary rhythms over their whole extent, there is no reason whatsoever to expect the appearance of an intermediate tone. This tone cannot, of course, be due to the sympathetic vibration of the undertone fibres, since these give the same sensation as the primary strips.[3]

As regards Obs. (*a*), therefore, Ebbinghaus leaves us just where we were left by Helmholtz. It is as if Stumpf had never written.

We pass to Obs. (*b*). "Aber es [the Helmholtz explanation] passt nicht mehr," Ebbinghaus goes on, "bei grösseren Entfernungen der primären Töne, etwa von einem Halbton ab.

---

[1] *Ibid.*, 317.

[2] Were it not for Helmholtz' specific energies, his theory might easily be so extended as to cover the intermediate tone of Obs. (*a*); for under certain conditions the superposition of two pendular vibrations of nearly the same frequency gives rise, theoretically, to a resultant vibration which lies midway between. See Rayleigh; *Theory of Sound*, 2 ed., i, 1894, 49,71; ii, 1896, 443, 450. In confirmation of these theoretical deductions, R. H. M. Bosanquet (*Phil. Mag.*, xi, 5 ser., 1881, 421) actually heard both the intermediate tone of Obs. (*a*) and the beating mass of Obs. (*c*) as lying midway between the primaries. Now Helmholtz' theory of analysis makes the beats of Obs. (*c*) mark the limit of Ohm's law; *i. e.*, beats arise when analysis is replaced by interference. The modification offered by the evidence just cited simply sets the limit of the law one degree higher, and brings not only the beating mass of (*c*) but also the intermediate tone of (*a*) under interference.

[3] Having broken away from H.'s specific energies, E. might well have explained the *Zwischenton* on the physical basis of the preceding note. But he rests on H.'s explanation, which is plainly inadequate.

Dann verschwindet der Zwischenton vollkommen, die Schwebungen aber, die noch an ihm haften sollen, verschwinden nicht auch, sondern bleiben hörbar noch bei beträchtlich grösseren Intervallen, und zwar haften sie deutlich an den beiden primären Tönen. Als Träger der Schwebungen können mithin, wie die Sache auch oben bereits dargestellt wurde,[1] keine anderen Fasern der Basilarmembran in Betracht kommen als die den schwebenden Tönen direkt zugeordneten. Und nur wenn diese bis auf einen Halbton und weniger aneinander rücken, kommt es zu einem für die Empfindung merklich werdenden Uebereinandergreifen der beiden Erregungsgebiete."[2]

The vehicles of the beats are, in this case, not the intermediate overlapping fibres, but the fibres directly correlated with the stimulus rhythms. How do these fibres mediate beats? By virtue of their inelasticity. "Die mitschwingenden feinen Teilchen der Basilarmembran . . . haben doch zweifellos nur geringe elastische Kräfte, und können ihre Bewegung beim Authören des objektiven Anstosses aus sich heraus keine nennenswerte Zeit weiter fortsetzen. . . . Jene Amplitudenschwankungen der objektiven Tonwelle werden demnach von den sämmtlichen mit ihr schwingenden Gebieten der Basilarmembran leidlich getreu mitgemacht. Erfolgen sie relativ langsam, so hören wir sie als Schwebungen, bei grösserer Schnelligkeit als Rasseln, bei noch grösserer als Rauhigkeit."[3]

Now no one can deny that the basilar fibres may be as inelastic as Ebbinghaus supposes. But, if they are, their inelasticity should be taken into account for the other observations.[4] For inelasticity comes to the same thing as interference, since both mean a failure of analysis. If the analysis is incomplete in the larger intervals of (*b*), it must also be incomplete in the smaller intervals of (*a*), and the 'resting primaries' remain a mystery. In other words, E.'s appeal to inelasticity advances the limit of auditory analysis to the most remote separation of the primaries that gives rise to beats (*b*). The appeal must

---

[1] See p. 303.

[2] *Ibid.*, 317.

[3] *Ibid.*, 1902, 322; *cf.* 1897, 303. On p. 322 occurs the sentence: "Die mitschwingenden feinen Teilchen der Basilarmembran dürfen nun nicht etwa gedacht werden wie stimmgabelartige Gebilde, wie es seitens der Helmholtzschen Theorie eigentlich geschieht." How this can be said to 'eigentlich geschehen' on the part of the Helmholtz theory, in view of Helmholtz' own discussion of the damping of vibrations in the ear (*Sensations of Tone*, 142 *b* to 143 *d*), we fail to see.

[4] We have sought in vain, in Ebbinghaus' pages, for any definite qualification or limitation of this attribute of inelasticity. If it extends as far as beats extend, then we might surely look for fluctuations in the difference tones,—since they, like beats, depend upon an intensively fluctuating stimulus. *Cf.* also M. Meyer's remarks upon certain mistuned intervals. *Zeits.*, xvi, 1897, 15; Stumpf's *Beitr.*, ii, 1898, 39.

then *a fortiori* cover all lesser separations, and the smooth-sounding tones of (*a*) become anomalous.

Again, Ebbinghaus has, in terms of his own modification of the Helmholtz theory, an explanation of Obs. (*b*). The basilar strips directly correlated with the stimuli are so wide apart that there is no overlapping. But the undertone fibres are also vibrating; and the interval between these fibres gets steadily smaller and smaller, as the fibres themselves become longer. A point must be reached, before very long, at which the undertone strips will overlap, and at which there will consequently be beats due to Helmholtzian interference. There is, thus, no necessity for the appeal to inelasticity. The beats of Obs. (*b*) may be due to phenomena of interference among the undertone fibres. Ebbinghaus himself makes no mention of this possibility.

Our criticism of Ebbinghaus, then, amounts to this: that he has multiplied causes for Obs. (*b*), and has altogether failed to account for the intermediate tone and the resting primaries of Obs. (*a*). We do not say how far interference is due to inelasticity; we do not know. But if the fibres of the basilar membrane are inelastic, there is no place for the smoothly sounding tones of the latter Obs. —

Although the matter of beats is far from settled, the writers venture to suggest that the case is not altogether hopeless. In regard to the three introspective observations, they offer the following considerations. Obs. (*c*), the beating mass, is probably correlated with cochlear vibrations which repeat with more or less fidelity the resultant from two pendular vibration rates. Obs. (*a*) depends, in part, upon two analyzed components; and in part, upon an unanalyzed residue possessing an intermediate frequency. Obs. (*b*) is, finally, to be referred to an almost complete analysis, in the sense of Ohm's law. The beating of the primaries may be explained by a small area of interference,—an area insufficient to mediate a distinct tone. The argument for Obs. (*b*) is supported by three bits of evidence. First, in Stumpf's introspections, it was difficult to localize the beat: the two primaries '*seem* themselves to beat;' but if either is attended to, that one '*always appears* to be the beating tone.' Secondly, weak and circumscribed interference might well give rise to intensive fluctuations without, however, producing a distinct tone (*cf.* weak partials discoverable only by beats). Thirdly, in our own experience, tones (*e. g.*, g^1 and a^1) which, when weak, are heard beating separately, give when excited more intensively a suggestion of an additional intermediate tone.